MODEL QUESTION PAPER

M.Sc. Mathematics Second year

MScMD2.01 COMPLEX ANALYSIS

Answer any THREE questions
All questions carry equal marks

(3 x 10 = 30)

1. (a) Show that an analytic function with constant modulus in constant.

   (b) State and prove Cauchy Riemann equations in polar co-ordinates.

2. (a) Show that every Mobius transformation $W = L(z) = \frac{az+b}{cz+d}(ad - bc \neq 0)$ is circle preserving.

   (b) Find the Mobius transformation which carries the points -1, I, 1+I into 0, 2i, 1-i

3. State and prove Cauchy's integral theorem for triangles using the key lemma.

4. (a) State and prove cauch's integral formula.

   (b) State and prove the Morera's theorem.

5. (a) State and prove Cauchy Hadamard theorem.

   (b) Find the Radius of convergence of the power series $\sum_{n=1}^{\infty} x^n z^n$.

6. (a) Let f(z) be an analytic on a domain G, let z_0 be an arbitrary point of G, and let $\Delta = \Delta(z_0)$ be the distance between z_0 and the boundary of G. Then there exists a power series $f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$ converging to f(z) on the disk $K : |z - z_0| < \Delta$.

7. (a) State Laurent's theorem.

   (b) Expand the function $f(x) = \frac{z^2 - 2z + s}{(z-2)(z^2+1)}$ is a Laurent series on the annulus $1 < |z| < 2$.

(c) Find the singular points and investigate the behavior at infinity of the function $(z)=\frac{1}{z-z^3}$.

8. (a) State and prove residue theorem

(b) Find the residues of $f(x)=\frac{1}{z(1-z^2)}$ at all its isolated singular points.

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MODEL QUESTION PAPER

M.Sc. Mathematics Second year

MScMD2.02 FUNCTIONAL ANALYSIS

Answer any THREE questions
All questions carry equal marks

(3 x 10 = 30)

1. State and prove the Hahn-Banach theorem.

2. State and prove the open mapping theorem.

3. (a) Show that every non-zero Hilbert space contains a contains a complete orthornomal set.

   (b) If M is a closed linear subspace of a Hilbert space H, then prove that $H = M \oplus M^1$.

4. (a) If T is an operator on H for whish $(Tx, x) = 0$ for all x, then prove that $T = 0$.

   (b) If $\underline{P}$ is the projection on a closed linear subspace M of H, then prove that M is invariant under an operator $T \Leftrightarrow TP = PTP$.

5. State and prove the spectral theorem.

6. (a) Let A be banach algebra. Show that every maximal left ideal in A is closed.

   (b) Show that A/R is a semi-simple banach algebra.

7. State and prove the Gelfand Newmark theorem.

8. If X is a compact Housdorff space, then prove that every closed ideal in (X) is the intersection of the maximal ideals which contain it.

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MODEL QUESTION PAPER

M.Sc. Mathematics Second year

MScMD2.03 MATHEMATICAL METHODS

Answer any THREE questions
All questions carry equal marks

(3 x 10 = 30)

1. Solve the integral equation $y(x)=1+\int_0^x y(t)\sin(x-t)\,dt$.

2. Solve the equation $y(x)1+\lambda\int_{-x}^{x} e^{iw(x-1)}y(t)\,dt$ considering separately all exceptional cases.

3. (a) Find the Laplace transform of $\sin\sqrt{t}$.

   (b) Find $L\{tj_0(at)\}$.

4. (a) Prove that $\int_0^t J_0(u)J_0(t-u)\,du=\sin t$.

   (b) Find $L^{-1}\left\{\frac{P}{\left(P^2+Q^2\right)^2}\right\}$ by using convolution theorem.

5. Using Laplace transform solve:
   $Dx+Dy=t$
   $D^2x-y=e^{-t}$ with $x(0)=3, x'(0)=-2, y(0)=0$.

6. (a) Solve $F'(t)=t+\int_0^t F(t-u)\cos u\,du,\ f(0)=4$.

   (b) Solve $\int_0^t \frac{F(u)}{\sqrt{t-u}}\,du=1+t+t^2$.

7. (a) Solve the integral equation
   $\int_0^\infty f(x)\cos\lambda x\,dx=e^{-2}$.

   (b) Find the cosine transform of e^{-x^2}.

8. (a) Use Fourier integral, show that $e^{-ax}=\frac{2a}{\pi}\int_0^\infty \frac{\cos\lambda x}{\lambda^2+a^2}\,d\lambda, a>0, x\geq 0$

   (b) Find finite Fourier sine transform of $f(x)=x^2, 0<x<4$.

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MODEL QUESTION PAPER

M.Sc. Mathematics Second year

MScMD2.04 NUMERICAL ANALYSIS

Answer any THREE questions
All questions carry equal marks

(3 x 10 = 30)

1. (a) The table gives the values of tan z for $0.10 \le x \le 0.30$.

x	0.10	0.12	0.20	0.25	0.30
y=tan x	0.1003	0.1511	0.2027	0.2553	0.3093

Find (i) tan 0.12 (ii) tan 0.26.

(b) Certain corresponding values of x and $\log_{10} x$ are (300, 2.4771), (304, 2.4829), (305, 2.4843) and (307, 2.4871). Find $\log_{10} 301$.

2. (a) Fit a natural cubic spline to the following data:

X	1	2	3
Y	-8	-1	18

And compute(i) y(1.5) and (ii) $y'(1)$.

(b) Derive an expression for the transaction error in the cubic spline.

3. (a) Find, from the following table, the area bounded by the curve and the x-axis from x = 7.47 to x = 7.52.

X	7.47	7.48	7.49	7.50	7.51	7.52
F(x)	1.93	1.95	1.98	2.01	2.03	2.06

(b) Use Romberg's method to compute $I = \int_0^1 \frac{1}{1+x} dx$, correct to three decimal places.

4. Solve the equations.

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$
$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$
$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$
$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

Using (a) Gauss – Seidel method (b) Jacobi's method.

5. (a) Given $\frac{dy}{dx} = y - x$ where $y(0) = 2$, find $y(0.1)$ and $y(0.2)$ correct to four decimal places using Runge – Kutta fourth order formula.

(b) Consider the differential equation $y' = -y$ with the condition y(0) = 1. Using Euler's method, find y(0.04), taking h=0.01.

6. Given $\frac{dy}{dx} = 1 + y^2$. Where y = 0 when x = 0. Compute y(0.8) using Adams – Moulton method.

7. (a) From the Taylor series for y(x), find y(0.1) correct to four decimal places if y(x) satisfies $y' = x - y^2$ and y(0) =1.

(b) Explain Picard's method of successive approximations.

8. Explain the method of characteristics and derive a solution of $\frac{\delta^2 u}{\delta x^2} - u^2 \frac{\delta^2 u}{\delta y^2} = 0$ at the grid point between x = 0.2 and 0.3, y>0 where y satisfies the conditions $u = 0.2 + 5x^2$ and $\frac{\delta u}{\delta y} = 3x$ along the initial line y = 0 for $0 \le x \le 1$.

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MODEL QUESTION PAPER

M.Sc. Mathematics Second year

MScMD2.05 FLUID MECHANICS(Elective)

Answer any THREE questions
All questions carry equal marks

(3 x 10 = 30)

1. Derive the continuity equation and test the whether the motion (is a constant) is a possible motion for an incompressible fluid. If so, find the streamlines equations.
2. (a) Discuss the pressure at a point in a fluid at rest and in a moving fluid.
   (b) Derive the Bernoulli's equation of motion.
3. Discuss the problem of underwater explosion giving spherical gas bubble.
4. (a) Discuss the impulsive motion.
   (b) Derive the Stokes's stream function.
5. Derive the stream function and discuss it for two-dimensional incompressible flow.
6. State and prove Blasius theorem.
7. (a) Discuss the translational motion of fluid element.
   (b) Derive the relation between stress and rate of strain.
8. (a) Derive the stress analysis in fluid motion.
   (b) Derive the Navier-Stokes equations of motion of a viscous fluid.

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MODEL QUESTION PAPER

M.Sc. Mathematics Second year

MScMD2.05 GRAPH THEORY(Elective)

Answer any THREE questions
All questions carry equal marks

(3 x 10 = 30)

1. (a) Define the degree of a vertex, hipartite graph. Illustrate with examples(Two examples atleast)

   (b) Show that if G is a simple graph then $e \le \binom{v}{2}$.

2. (a) With the usual notation show that $\delta \le \frac{2e}{v} \le \Delta$

   (b) Prove that in any graph the number of vertices of odd degree is even.

3. (a) Define a tree and illustrate with an example. Prove that every non trivial tree has atleast two vertices of degree one.

   (b) Prove that a connected graph is a tree if and only if every edge is a cut edge.

4. (a) With the usual notation, show that $\Re \le \Re' \le \delta$.

   (b) Show that $K\left(H_{m,n}\right) = \Re'\left(H_{m,n}\right) = m$.

5. (a) Define an Euler graph. Prove that a necessary and sufficient condition for a nonempty connected graph in eulerian if and only if it has no vertices of odd degree.

   (b) Define the closure of a graph. Prove that a graph is Hamiltonian if and only if its closure is Hamiltonian.

6. (a) Define a perfect matching in a graph. Prove that if G is a $\Re$- regular bipartite graph with $\Re > 0$ then G has a perfect matching.

   (b) If G is a bipartite graph, with the usual notation prove that $X' = \Delta$.

7. (a) What is an independent of a graph : Give an example. With the usual notation, prove that $\alpha + \beta = \nu$.

(b) For any two integers $\Re \geq 2, l \geq 2$ prove that $r(\Re, l) \leq (\Re, l-1) + r(\Re - 1, l)$.

8. (a) Prove that $r(\Re, R) \geq 2^{\frac{k}{2}}$.

(b) Describe how the Hungarian method can be used to find a maximum matching in a bipartite graph.

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