SRI PADMAVATI MAHILA VISVAVIDYALAYAM, TIRUPATI

(WOMEN'S UNIVERSITY)

MASTER OF SCIENCE (MATHEMATICS)

Assignment Question Paper

Paper : M.Sc. M.D 1.01 — ALGEBRA

Answer any Three questions.

All questions carry equal marks.

3X10=30

1. (a) Express the following permutation as a product of disjoint cycles : $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 1 & 6 & 4 & 3 \end{pmatrix}$.

   (b) State and prove second Sylow theorem.

2. (a) If G is a group of order pq where p and q are distinct primes and if G has a normal subgroup H of order p and normal subgroup K of order q, then prove that G is cyclic.

   (b) If the order of a finite group G is divisible by a prime number p, then prove that G has an element of order p.

3. (a) Define characteristic of a ring with an example.

   (b) Prove that a finite integral domain is a field.

4. (a) Prove that a homomorphism ϕ of R into R' is an isomorphism if and only if Kernel $I(\phi) = (0)$.

   (b) If U is an ideal of a ring R, then prove that the ring R/U is a homomorphic image of R.

5. Prove that every integral domain can be imbedded in a field.

[P.T.O.]

M S Parvathi

6. (a) Prove that an ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R if a_0 is a prime element of R.

(b) If R is an integral domain, the prove that $R[x]$ is also an integral domain. (7+7)

7. (a) Prove that $L(S)$ is a subspace of V.

(b) If $v_1, v_2, \ldots, v_n$ are in a vector space v. Then prove that they are linearly independent or some v_k is a linear combination of the preceding ones, $v_1, v_2, \ldots, v_{k-1}$.

8. (a) If V is finite dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V on to V.

(b) If $\lambda \in F$ is a characteristics root of $T \in A(V)$, then prove that λ is a root of the minimal polynomial of T.

H Sarvath

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Master of Science (MATHEMATICS)

Assignment Question Paper

Paper : M.Sc. MD 1.02 — REAL ANALYSIS

3x10=30.

Answer any Three questions.

All questions carry equal marks.

1. (a) Define Metric space. For $x \in R$ and $y \in R$, $d(x, y) = (x - y)^2$; determine Whether it is a metric or not.

   (b) Prove that every neighbourhood is an open set.

2. (a) Prove that compact subsets of metric spaces are closed.

   (b) Prove that closed subsets of compact sets are compact.

3. (a) Let 'f' and 'g' be complex continuous functions on a metric space X. Then prove that $f + g$, fg and f / g are continuous on X.

   (b) If p^* is a refinement of p, then show that $L(p, f, \alpha) \le L(p^*, f, \alpha)$ and $U(p^*, f, \alpha) \le U(p, f, \alpha)$.

4. (a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that 'f' is uniformly continuous on X.

   (b) Let 'f' be monotonic on (a, b). Then prove that the set of points of (a, b) at which 'f' is discontinuous are atmost countable.

5. (a) Let 'f' be defined on $[a, b]$. If 'f' is differentiable at a point $x \in [a, b]$ then show that 'f' is continuous at X.

   (b) Suppose 'f' is differentiable in (a, b). If $f'(x) \ge 0 \; \forall x \in (a, b)$ then prove that 'f' is monotonically increasing.

[P.T.O.]

M Savath

6. (a) Suppose 'f' is a continuous mapping of $[a,b]$ into R^K and 'f' is differentiable in (a,b). Then show that there exits $x \in (a,b)$ such that

$$|f(b) - f(a)| \le (b-a)|f'(x)|.$$

(b) If $a < s < b$, 'f' is bounded on $[a,b]$, 'f' is continuous at s and $d(x) = I(x-s)$, then show that $\int_a^b f\,dx = f(s)$.

7. State and prove Lebesgue Monotone convergence theorem.

8. (a) Prove that For every $A \subset R$, $\mu^*(A) = \mu(A)$.

(b) If $E = \bigcup_{n=1}^{\infty} E_n$; then show that $\mu^*(E) \le \sum_{n=1}^{\infty} \mu^*(E_n)$.

M Sfaualto

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MASTER OF SCIENCE (MATHEMATICS)

Assignment Question Paper

Mathematics

Paper M.Sc. MD 1.03 – DIFFERENTIAL EQUATIONS

Answer any Three questions.

All questions carry equal marks.

3 × 10 = 30.

1. (a) Define the following :
   (i) Ordinary point
   (ii) Singular point
   (iii) Regular singular point
   (iv) Radius of convergence with example.

   (b) Find the power series solution of the differential equation $y' = y$ interms of power series in 'x'.

2. (a) Find two independent Frobenius series solutions of the equation

$$x^3(x-1)y'' - 2(x-1)y' + 3xy = 0$$

   (b) Find the regular singular points of the eqn $x^2y'' + xy' + (x^2 - p^2)y = 0$.

3. (a) Prove that $P_n(x) = \dfrac{1}{2^n \cdot n!}\dfrac{d^n}{dx^n}(x^2-1)^n$.

   (b) Show that $\int_{-1}^{1} P_m(x) \cdot P_n(x)\, dx = \begin{cases} 0 & if\ m \neq n \\ \dfrac{2}{2n+1} & if\ m = n \end{cases}$.

[P.T.O.]

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4. (a) Prove that $J_{1/2}(x) = \sqrt{\dfrac{2}{\pi x}} \sin x$ and $J_{-1/2}(x) = \sqrt{\dfrac{2}{\pi x}} \cos x$.

(b) Show that $\left(n + \dfrac{1}{2}\right)! = \dfrac{(2n+1)!}{2^{2n+1} \cdot n!} \sqrt{\pi}$.

5. (a) Find the exact solution of the IVP $y' = 2x(1+y)$, $y(0) = 0$ starting $y_0(x) = 0$.

(b) Show that $f(x, y) = y^{1/2}$ does not satisfy a Lipschitz condition on the rectangle $|x| \le 1$ and $0 \le y \le 1$.

6. (a) Find the general integral of the linear PDE $y^2 p - xyq = x(z - 2y)$.

(b) Find the complete integral of $(p^2 + q^2)y - qz = 0$.

7. (a) Find the solution of the equation $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial y^2} = x - y$.

(b) Solve $(D^2 + 2DD' + D'^2)y = e^{2x+3y}$.

8. (a) Reduce the equation $\dfrac{\partial^2 z}{\partial x^2} + 2\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2} = 0$ to canonical form and solve it.

(b) Solve $(D^2 - D^1)z = e^{x+y}$.

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MASTER OF SCIENCE (MATHEMATICS.

Assignment Question Paper

PAPER : MSCMD 1.04 — LINEAR PROGRAMMING

Answer any Three questions.

All questions carry equal marks.

3×10=30.

1. (a) What is operations research? State the different types of models used on operations research.

   (b) Write the procedure for Mathematical formulation of the Linear Programming Problem.

2. (a) Solve the LPP graphically :

   Maximize $Z = 3x_1 + 2x_2$

   Subject to constraints : $-2x_1 + x_2 \le 1$,
   $x_1 \le 2$,
   $x_1 + x_2 \le 3$,
   $x_1, x_2 \ge 0$.

   (b) Write the standard form of L.P.P. in matrix from.

[P.T.O.]

C.V. Lal

3. (a) Use simplex method to solve the L.P.P.

Maximize : $Z = 3x_1 + 2x_2$

Subject to the constrains $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$;

$x_1 \geq 0$,

$x_2 \geq 0$.

(b) Explain the following terms :

(i) Slack variables

(ii) Surplus variables

(iii) Artificial variables.

4. (a) Use two phase simplex method to

Maximize : $Z = 3x_1 + 2x_2$

Subject to constraints $2x_1 + x_2 \leq 2$,

$3x_1 + 4x_2 \geq 12$;

$x_1, x_2 \geq 0$.

(b) What is the problem of degeneracy?

5. (a) Obtain the dual problem of the following L.P.P.

Maximize $f(x) = 2x_1 + 5x_2 + 6x_3$

Subject to $5x_1 + 6x_2 - x_3 \leq 3, -2x_2 + x_2 + 4x_3 \leq 4$

$x_1 - 5x_2 + 3x_2 \leq 1, \ -3x_1 - 3x_2 + 7x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$.

(b) State and prove complementary slackness theorem.

6. Use dual simplex method to solve the following L.P.P. :

Maximize $z = -3x_1 - x_2$

Subject to $x_1 + x_2 \geq 1, \ 2x_1 + 3x_1 \geq 2, x_1, x_2 \geq 0$.

C.V. Lal

7. (a) Obtain an initial basic feasible solution to the following transformation problems using matrix maxima method or least cost method. .

	D_1	D_2	D_3	D_4	Supply
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	

(b) Explain unbalanced transformation problem.

8. (a) Give a mathematical formulation of the assignment problem.

(b) Solve the following travelling salesman problem so as to minimize the cost per cycle :

	To				
From	A	B	C	D	E
A	–	3	6	2	3
B	3	–	5	2	3
C	6	5	–	6	4
D	2	2	6	–	6
E	3	3	4	6	–

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(WOMEN'S UNIVERSITY)

MASTER OF SCIENCE (MATHEMATICS)

Assignment Question Paper

Paper MScMD 1.05 — TOPOLOGY

Answer any Three of the following.

All questions carry equal marks.

$3 \times 10 = 30$

1. (a) Let X be a topological space. If A and B are arbitrary subsets of X, show that

   (i) $\overline{\phi} = \phi$

   (ii) $A \subseteq \overline{A}$

   (iii) $\overline{\overline{A}} = \overline{A}$ and

   (iv) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

   (b) Let X be a topological space and A an arbitrary subset of X. Show that $\overline{A} = (x /$ each neighbourhood of x intersects $A\}$.

2. (a) State and prove lindelof's theorem.

   (b) Show that every separable metric space is second countable.

3. (a) Show that a topological space is compact if every basic open cover has a finite subcover.

   (b) Show that the product of any non-empty class of compact spaces is compact.

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