SRI PADMAVATI MAHILA VISVAVIDYALAYAM, TIRUPATI

(WOMEN'S UNIVERSITY)

MSC Mathematics II year

Assignment question Paper

Paper : M.Sc.MD 2.01 — COMPLEX ANALYSIS

Answer any Three questions.

All questions carry equal marks.

3 × 10 = 30

1. (a) Derive Cauchy-Riemann Equations in Polar Coordinates.

   (b) Show that the function $f(z) = \sqrt{|xy|}$ satisfies the Cauchy-Riemann equations at the point $z = 0$, but is not differentiable there.

2. (a) Show that every Mobius transformation different from the unit transformation has two fixed points, which in certain cases coalesce into a single fixed point.

   (b) Find the Mobius transformation which carries the points -1, ∞, i into the points ∞, i, 1.

3. Evaluate the Fresnel integrals $\int_0^\infty \cos x^2 dx$, $\int_0^\infty \sin x^2 dx$.

4. (a) If $f(z)$ is analytic on a domain G, show that $f(z)$ has derivatives of all orders on G, and in fact, given any $z_0 \in G$, $f^n(z_0) = \frac{n!}{2\pi i}\int \frac{f(z)}{(z-z_0)^{n+1}} dz$ $(n = 0, 1, 2, \ldots)$, where L is any closed rectifiable Jordan curve such that $z_0 \in I(L)$, $\overline{I(L)} \subset G$.

C.V. L

(b) Let $f(z)$ be continuous on a domain G and suppose than $\int_L f(z)\,dz = 0$ for every closed rectifiable curve L contained in G. Show that $f(z)$ is analytic on G. (5)

5. State and prove Cauchy-Hadamard Theorem. (14)

6. (a) Find the radius of convergence of ~~each of~~ the series $\sum_{n=1}^{\infty} \frac{n^k}{n!} z^n \; (k = 0, 1, 2 ...)$.

(b) Expand the function $\frac{z^2}{(z+1)^2}$ in Taylor series at the point $z = 1$. (7 + 7)

7. (a) Let $f(z)$ be an analytic function on an annulus $D : r < |z - z_0| < R'$. Show that there exists a Laurent series $f(z) = \sum_{n=-\infty}^{\infty} a_n (z - z_n)^n$ converging to $f(z)$ on D. (9)

(b) Expand the function $f(z) = \frac{1}{z(1-z)}$ in a Laurent series at $z = 0$ and $z = 1$ on the annulus $0 < |z| < 1$. (5)

8. Evaluate the integral $\oint = \int_0^{\infty} \frac{\cos \lambda x}{x^2 + a^2} dx = \frac{\pi e^{-\lambda a}}{2a} (a > 0, \lambda > 0)$. (14)

C.V. Lal

SRI PADMAVATI MAHILA VISVAVIDYALAYAM, TIRUPATI

(WOMEN'S UNIVERSITY)

[illegible])

Assignment Question Paper

Paper : M.Sc.MD 2.02 — FUNCTIONAL ANALYSIS

Answer any Three questions.

All questions carry equal marks.

3 X 10 = 30

1. Let N and N' be normed linear spaces and T a linear transformation of N into N'. Show that the following conditions on T are all equivalent to one another :

   (a) T is continuous;

   (b) T is continuous at the origin, in the hence that $x_n \to 0 \Rightarrow T(x_n) \to 0$;

   (c) there exists a real number $K \geq 0$ with the property that $\|T(x)\| \leq K\|x\|$ for every $x \in N$;

   (d) if $S = \{x : \|x\| \leq 1\}$ is the closed unit sphere in N, then its image $T(S)$ is a bounded set in N'.

2. Let M be a linear subspace of a normed linear space N, and let f be a functional on M. If x_0 is a vector not in M, and if $M_0 = M + [x_0]$ is the linear subspace spanned by M and x_0, show that f can be extended to a functional f_0 defined on M_0 such that $\|f_0\| = \|f\|$. Hence show that f can be extended to a functional f_0 defined on the whole space N such that $\|f_0\| = \|f\|$.

[illegible signature]

3. (a) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

(b) Let M be a closed linear subspace of a Hilbert space H, let x be a vector not in M, and let d be the distance from x to M. Show that there exists a unique vector y_0 in M such that $\|x - y_0\| = d$.

4. (a) If A is a positive operator on a Hilbert space H, prove that $I + A$ is non- singular. In particular prove also that $I + T^*T$ and $I + TT^*$ are non-singular for an arbitrary operator T on H.

(b) If T is on an operator on a Hilbert space H, show that T is normal $\Leftrightarrow$ its real and imaginary parts commute.

5. (a) If T is normal, show that x is an eigenvector of T with eigenvalue $\lambda \Leftrightarrow x$ is an eigenvector of T^* with eigenvalue $\overline{\lambda}$.

(b) If T is normal, show that the M_i's are pairwise orthogonal.

6. (a) Let G be the set of regular elements in a Banach Algebra A and let x be an element of G. Show that the mapping $x \to x^{-1}$ of G into G is continuous and is therefore a homeomorphism of G onto itself.

(b) Let S be the set of singular elements in a Banach Algebra A and let Z be the set of all topological divisors of zero. Prove that the boundary of S is a subset of Z.

7. (a) If 0 is the only topological divisor of zero in a Banach Algebra A, show that $A = C$.

(b) Define the spectral radius $r(x)$ of an element x in a Banach Algebra A and show that $r(x) = \lim \|x^n\|^{1/n}$.

8. State and prove Gelfand-Neumark theorem.

SRI PADMAVATI MAHILA VISVAVIDYALAYAM, TIRUPATI

(WOMEN'S UNIVERSITY)

MASTER OF SCIENCE (MATHEMATICS)

Assignment Question Paper

Paper MScMD 2.03 — MATHEMATICAL METHODS

Answer any Three questions.

Each question carries 14 marks.

3 x 10 = 30

1. Solve the equation $y(t) = 1 + \lambda \int_{-x}^{x} e^{iw(x-t)}\, y(t)\, dt$ considering separately all exceptional cases.

2. (a) Find the Laplace transform of the function $f(t) = \dfrac{\sinh t}{t}$.

   (b) Solve $L[e^{-at} J_0(at)]$.

3. (a) Prove that $L^{-1}\left\{\dfrac{f(s)}{s}\right\} = \int_0^t F(u)\, du$ and based on this result solve $L^{-1}\left\{\dfrac{1}{s^2(s+1)^2}\right\}$.

   (b) Show that $\int_0^\infty \cos x^2\, dx = \dfrac{1}{2}\sqrt{\dfrac{\pi}{2}}$.

4. Apply convolution theorem to show that $\int_0^t \sin u \cos(t-u)\, du = \dfrac{1}{2}\sin t$.

[P.T.O.]

C. V. Ld

5. (a) Apply Laplace transform to solve $\frac{d^2y}{dx^2} + y = 6\cos 2t$ given that $y = 3$, $\frac{dy}{dt} = 1$ when $t = 0$.

(b) Solve $(D^2 + 3D + 2)x = 0$ and $x = x_0, D_x = x$ at $t = 0$.

6. (a) Solve the integral equation $F(t) = 1 + \int_0^t F(u)\sin(t-u)\,du$.

(b) Prove that $\int_0^t \frac{F(u)}{\sqrt{t-u}}\,du = 1 + t + t^2$.

7. (a) Find the Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin s}{s}\,ds$.

(b) Find the Fourier cosine transform of e^{-x^2}.

8. (a) Find the sine transform of $\frac{x}{1+x^2}$.

(b) Find $f(x)$, if the sine transform is $\frac{e^{-as}}{s}$.

C.V. Ld.

SRI PADMAVATI MAHILA VISVAVIDYALAYAM, TIRUPATI

(WOMEN'S UNIVERSITY)

MASTER OF SCIENCE (MATHEMATICS)

Assignment Question Paper.

Paper : MSC MD 2.04 — NUMERICAL ANALYSIS

Answer any Three questions.

All questions carry equal marks.

3 × 10 = 30

1. (a) Find the cubic polynomial which takes the following values $y(0)=1$, $y(1)=0$, $y(2)=1$ and $y(3)=10$. Hence or otherwise obtain $y(4)$.

   (b) Given $x=1,2,3,4$ and $f(x)=1,2,9,28$ respectively find $f(3.5)$ using Lagranges method of 2nd and 3rd order degree polynomials.

2. (a) Find $\frac{dy}{dx}$ at $x=7.5$ from the following table :

x:	7.47	7.48	7.49	7.5	7.51	7.52	7.53
y:	0.193	0.195	0.198	0.201	0.203	0.206	0.208

   (b) Using the following data , find x for which y is minimum and find this value of y.

x:	0	2	4	6
y:	3	3	11	27

[P.T.O.]

C.V. Lal

3. (a) Find the value of $\int_1^2 \frac{dx}{x}$ by $\frac{1}{3}$ Simpson's rule. Hence obtain approximate value of $\log_e^2$.

(b) Find the eigen values and the corresponding eigen vectors of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

4. (a) Using Taylor series method, solve the equation $\frac{dy}{dx} = x^2 + y^2$ for $x = 0.4$ given that $y = 0$ when $x = 0$.

(b) Using modified Euler method find $y(0.2)$ and $y(0.4)$ given $y' = y + e^x$, $y(0) = 0$.

5. (a) Apply fourth order R-K method to find $y(0.1)$ and $y(0.2)$ given $y' = xy + y^2$, $y(0) = 1$.

(b) Find $y(0.1)$ and $z(0.1)$ from the system of equations $y' = x + z$, $z' = x - y^2$ given $y(0) = -2$, $z(0) = 1$ using fourth order R-K method.

6. (a) Use Milne's method to find $y(0.8)$ and $y(1.0)$ given $y' = \frac{1}{x+y}$, $y(0) = 2$ and $y(0.2) = 2.0933$, $y(0.4) = 2.1755$ $y(0.6) = 2.2493$.

(b) Solve $x''(t) + x(t) = 6\cos t$ with $x(0) = 2, x'(0) = 3$ by using R-K 4th order method.

7. (a) Use the finite difference method solve $u_{tt}(x,t) = 4\,u_{xx}(x,t)$ for $0 \le x \le 1$ and $0 \le t \le 0.5$ with boundary conditions $u(0,t) = 0$ and $u(1,t) = 0$, $u(x,0) = \sin \pi x$ $u_t(x,0) = 0$ with $h = 0.2, k = 0.1$.

(b) Explain Crank-Nicholson method.

C.V. Lal